The Basic Problem Set

Problem 1. A track and field athlete was able to increase his speed by 1.5 times while running the last 20 m of the distance in 3.2 seconds. Find his speed at the end of the distance. Consider the movement of the athlete to be uniformly accelerated and rectilinear. Give the answer in [m/s] accurate to the nearest tenth.

Solution: Let the athlete's velocity increase from v_0 to v_1 in $t = 3.2$ s. Since the athlete moves with uniform acceleration, the final velocity will be $v_1 = v_0 + at$, where a is the acceleration of the athlete. We also know from the problem statement that

$$v_0 = \frac{v_1}{1.5} = \frac{2}{3}v_1.$$

Then

$$a = \frac{v_1}{3t}.$$

In time t, the athlete will run the path

$$S = \frac{v_1^2 - v_0^2}{2a}.$$

Substituting the previous expressions for the initial speed and acceleration into the last equation, we obtain

$$S = \frac{5}{6} \cdot v_1 t \implies v_1 = 1.2\frac{S}{t} = 1.2 \cdot \frac{20m}{3.2s} = 7.5\frac{m}{s}.$$

Answer: 7.5 m/s.

Problem 2. The tram motor operates at current $I = 90$ A and voltage $U = 400$ V. Moving evenly at $F = 4$ kN traction, the tram runs $S = 80$ m in a period of time $T = 10$ s. Find the resistance of motor winding. Give the answer in [Ohm] to an accuracy of tenth.

Solution: According to the energy conservation law, the electric current power in the motor winding $P_{el} = I \cdot U$ is greater than the traction power $P = F \cdot v = F \cdot \frac{S}{T}$ by the heat generation power I^2R in the motor winding, i.e.

$$I^2R = IU - F \cdot \frac{S}{T}.$$

Therefore,

$$R = \frac{1}{I}\left(U - \frac{F \cdot S}{I \cdot T}\right) = \frac{1}{90}\left(400 - 4 \cdot 10^3 \cdot \frac{80}{90 \cdot 10}\right) \approx 0{,}5 \text{ Ohm}.$$

Answer: 0.5 Ohm.

Problem 3. The iceberg rises above the ocean surface by $h_1 = 15$ m. Estimate the height of the iceberg taking its model in the form of a body with two flat horizontal bases and vertical side walls (a table-shaped iceberg). Ice density $\rho_1 = 900$ kg/m^3, seawater density $\rho_2 = 1028$ kg/m^3. Give the answer in [m] to the accuracy of the whole.

Solution: Let the height of the iceberg be h, the base area be S. Then the weight of the iceberg is $F = \rho_1 ghS$. The Archimedes force acting on the iceberg is equal to $F_A = \rho_2 g(h - h_1)S$. The iceberg equilibrium condition is that the Archimedes force is equal to the gravity force. That means,

$$\rho_1 ghS = \rho_2 g(h - h_1)S.$$

Then the sought quantity

$$h = \frac{\rho_2}{\rho_2 - \rho_1} h_1 = \frac{1028\ \text{kg/m3}}{1028\ \text{kg/m3} - 900\ \text{kg/m3}} \cdot 15m \approx 120m.$$

Answer: 120 m.

Problem 4. A piece of ice weighing 1 kg at a temperature of -20^{0}C is placed in the calorimeter containing two liters of water at a temperature of 40^{0}C. Determine the temperature in the calorimeter after establishing the thermal equilibrium. Give your answer in [^{0}C] accurate to the nearest whole.

Specific heat capacity of ice is 2100 J/(kg·^{0}C), specific heat capacity of water is 4200 J/(kg·^{0}C), specific heat of ice melting is 3.36·10^5 J/kg. Water density is 1000 kg/m^3. Heat capacity of the calorimeter negligible.

Solution: The final state is not obvious. Therefore, we must analyze the process.

To heat $m_\text{i} = 1$ kg of ice from $t_\text{i} = -20^\circ\text{C}$ to $t_0 = 0^\circ\text{C}$, the following amount of heat energy is required

$$Q_1 = c_\text{i} m_\text{i} (t_0 - t_\text{i}) = 42000\,\text{J},$$

where $c_\text{i} = 2100\,\text{J/(kg} \cdot {}^\circ\text{C)}$ is the heat capacity of ice.

Melting all the ice would require an amount of heat

$$Q_2 = \lambda_\text{i} m_\text{i} = 336000\ \text{J},$$

where $\lambda_\text{i} = 3.36 \cdot 10^5\ \text{J/kg}$ is the specific heat of ice melting.

If water in calorimeter is cooled down from $t_\text{w} = 40^\circ\text{C}$ to $t_0 = 0^\circ\text{C}$ then the following amount of heat will be released:

$$Q_3 = c_\text{w} \rho_\text{w} V_\text{w} (t_\text{w} - t_0) = 336000\ \text{J},$$

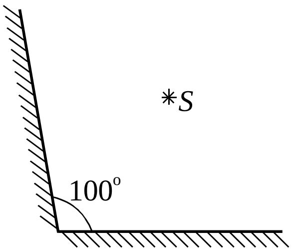
where c_w, ρ_w, and V_w are specific heat capacity, density, and volume of water, respectively.

Comparing the obtained values of Q_1, Q_2, and Q_3 we may conclude that Q_3 will be enough to heat the entire ice from t_i to t_0 and melt only a $\frac{7}{8}$ part of the ice.

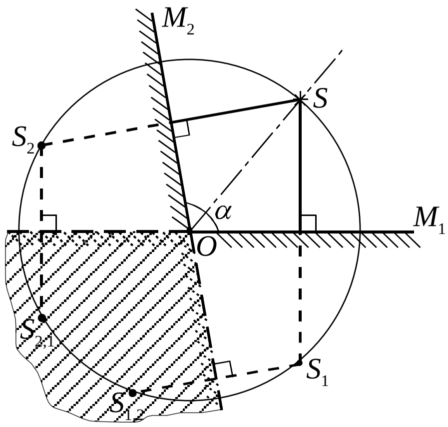
Therefore, the calorimeter will have a mixture of water and ice at temperature 0°C.

Answer: 0°C.

Problem 5. Two flat mirrors form an angle $\alpha = 100^\circ$ (see Fig.). On the bisector of this corner, there is a point light source. How many images of the source will produce such an optical system?



Solution: Since the point source S is located on the bisector of corner $\alpha = 100^\circ$, the system of mirrors will produce four images: the image S_1 of source S in the mirror M_1, image S_2 of source S in the mirror M_2, image $S_{1,2}$ of the image S_1 in the mirror M_2, and finally, image $S_{2,1}$ of the image S_2 in the mirror M_1. There will be no other images because the images $S_{1,2}$ and $S_{2,1}$ lie on the side of non-reflective surfaces of both mirrors (see Fig.). Note that all the images lie on a circle with the center at point O and radius $R = OS$.



Answer: four images.