Get It Solved 2019 Contest. Mathematics. Demo Version

Answers and Solutions

1. Some milk and coffee were mixed together in a big glass, the volume of coffee being equal to 84% of the total volume. After that some more coffee was added to the glass so that the volume of coffee became equal to 96% of the total volume. How much times is the new volume of new liquid greater than the initial one?

   Answer. 4.

   Solution. Let us denote the initial volume as x. It means that coffee constituted $0.84x$ out of it and milk constituted $0.16x$. After the coffee had been added, the volume of milk, that is $0.16x$, made only $100\% - 96\% = 4\%$ of the total volume y; hence, $0.16x = 0.04y$, and $y = 4x$.

2. All the students in a class are child prodigies. Each of the girls got 4 diplomas on school olympiads, 4 diplomas on city olympiads and 2 diplomas on regional olympiads, and each of the boys got 10 diplomas on school olympiads, 8 diplomas on city olympiads and 3 diplomas on regional olympiads. It is known that the amount of diplomas on city olympiads exceeds that on regional olympiads by 62. How many school olympiad diplomas did they get in the class?

   Answer. 124.

   Solution. Let g and b be the quantities of girls and boys in the class. Then the numbers of diplomas they got on school, city and regional olympiads are equal to $A = 4g + 10b$, $B = 4g + 8b$, $C = 2g + 3b$ respectively.

   It is known that $B - C = 62$, that is $(4g + 8b) - (2g + 3b) = 62$, $2g + 5b = 62$. Consequently, $A = 4g + 10b = 2(2g + 5b) = 2 \cdot 62 = 124$.

3. Legs AC and BC of a right triangle ABC are equal to 1 and 7 respectively. Angle bisector CL and altitude CH are drawn from the vertex of the right angle. Find the ratio of $CH : CL$. (Express the answer as a decimal fraction rounded up to one digit after the decimal point.)

   Answer. 0.8.

   Solution. By angle bisector theorem we get that $AL : BL = 1 : 7$, and therefore $BL : BA = 7 : 8$. Let P be a projection of L on BC. It means that $PL : AC = BL : BA$, and $PL = \frac{7}{8}$. In a right triangle CLP angle LCP is equal to $45°$; consequently, $CL = PL\sqrt{2} = \frac{7}{4\sqrt{2}}$. Pythagorean theorem for triangle ABC yields $AB = \sqrt{BC^2 + AC^2} = \sqrt{50} = 5\sqrt{2}$. Equating the doubled area of triangle ABC we get that $BC \cdot AC = CH \cdot AB$, or $7 \cdot 1 = CH \cdot 5\sqrt{2}$. Thus $CH = \frac{7}{5\sqrt{2}}$. Finally, $CH : CL = \frac{4}{5} = 0.8$.

4. A pedestrian and a cyclist start moving from point A to point B along a straight road. Each of them has a constant speed. As soon as the cyclist reaches B, he immediately turns around and goes back to A, turns around and goes back to B, and so on. While the pedestrian walks to B, he meets the cyclist several times. It is known that their first encounter has happened 15 minutes after they have started. How much time has passed before they meet for the fifth time (counting from the moment they started their motion)? Express the answer in minutes.

   Answer. 45.

   Solution. The total distance made by the pedestrian and the cyclist to the moment of their first meeting is equal to $2d$, where d is a distance between A and B. It implies that the time that passed before the first meeting is equal to $t = \frac{2d}{v}$ where v is sum of cyclist's and pedestrian's speeds. Their second meeting happened when the cyclist overtook the pedestrian, and their third meeting happened when the cyclist moved in the direction from B to A. Let us notice that the total distance made by the two between the first and the third encounters is equal to $2d$. It means that the time between the meetings equals t. In the same way we get that the time between the third and the fifth meetings is equal to t. All in all, they needed $3t = 45$ minutes until the fifth meeting.

5. A square with its side equal to 1 meter has been cut into several parts, the cuts being parallel to the sides of the square and going from one side of the square to another one. The total number of rectangles obtained this way is equal to 216. Find the smallest possible total length of all the cuts. Express the answer in meters.

Answer. 28.

Solution. Let us assume that we have made $a-1$ horizontal and $b-1$ vertical cuts, their total length being equal to $a+b-2$ meters. By these cuts the square is divided into ab rectangles. As $ab=216$, we conclude that 216 divides both a and b. Without loss of generality we can take $a \leq b$.

The equality $(a+b)^2=(b-a)^2+4ab=(b-a)^2+864$ shows that the minimum value of sum of lengths of all the cuts is reached for such pair of divisors a, b that their difference $b-a$ reaches its minimum value. There are no divisors of 216 between numbers 12 and 18, and $a=3\cdot 2^2=12$, $b=3^2\cdot 2=18$ satisfy the equation $ab=2^3\cdot 3^3=216$. Therefore, these are the values we need, and the minimum value of $a+b-2$ equals $12+18-2=28$.